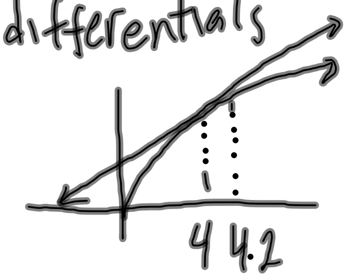
4.5 Linearizations, differentials

↑

tan lines



$y=\sqrt{x}$ linearization at $x=4$

$$y=\frac{1}{4}(x-4)+2$$

used in approximations

approximate $\sqrt{4.2}$: $\sqrt{4.2}\approx\frac{1}{4}(4.2-4)+2$

calc $\sqrt{4.2}=2.049$ $\frac{1}{4}(.2)+2=2.05$

Nov 2-12:58 PM

approx $\sqrt[3]{8.3}$ using linearizations

$y=\sqrt[3]{x}=x^{1/3}$ tan line at $x=8$

↑

close, easy

$y'=\frac{1}{3}x^{-2/3}$ $y'(8)=\frac{1}{3}8^{-2/3}$

$=\frac{1}{3}\cdot\frac{1}{4}=\frac{1}{12}$

tan line

$$y=\frac{1}{12}(x-8)+2$$

$$\sqrt[3]{8.3}\approx\frac{1}{12}(8.3-8)+2\approx 2.025$$

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$\cos(1.75)$ $y = \cos(x)$ we need an x that is close, easy

How about $x = \frac{\pi}{2} \approx 1.57\ldots$

$y' = -\sin x$

$y'\left(\frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{2}\right) = -1$

tan line eqn

$y = -1\left(x - \frac{\pi}{2}\right) + 0$

$\cos(1.75) \approx -\left(1.75 - \frac{\pi}{2}\right)$

$-.178 \approx -.179$

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$$\boxed{y(x) \approx y'(x_0)(x - x_0) + y_0}$$

easy & close to (x, y)

$$\boxed{\Delta y \approx y'(x_0)\,\Delta x}$$

change in y

change in x

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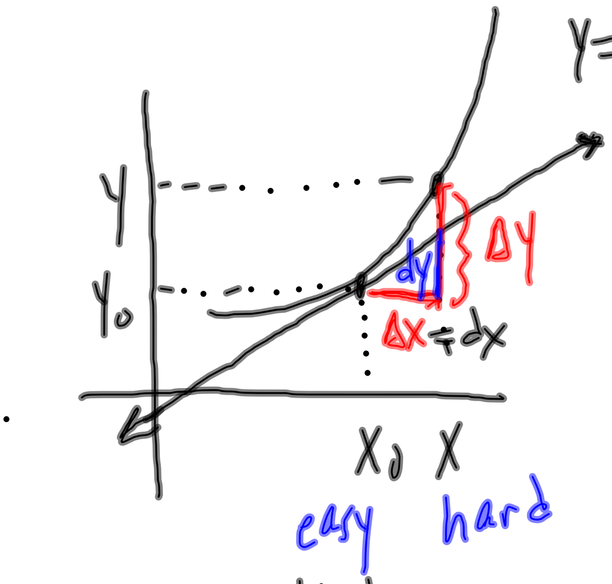

$\Delta x = dx$

$\Delta y \approx dy$

$$\boxed{dy = f'(x_0)\,dx}$$

dx : differential of x

dy : differential of y

$$\frac{dy}{dx} = f'(x_0)$$

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find the differential of tan(2x)

ie. $d(\tan(2x)) = 2\sec^2(2x) \cdot dx$

$\uparrow$

$$dy = \frac{dy}{dx} \cdot dx$$

Nov 2-1:46 PM

earth $r = 3959 \pm 0.1$ mi

(sphere) Δr

relative change in $r = \frac{\Delta r}{r}$

$= \frac{.1}{3959} = .000025$

percent change $= \frac{\Delta r}{r} \times 100\% = .0025\%$

estimate the percent change in surface area

$S = 4\pi r^2$

$\frac{\Delta y}{y} \times 100\% \approx \frac{dy}{y} \times 100\%$

$ds = 8\pi r \cdot dr$

$ds = 8\pi (3959)(0.1)$

$ds = 9950 \text{ mi}^2$

$\frac{9950}{4\pi \cdot 3959^2} \times 100\% = .005\%$

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